# DISCOVERY OF GRAVITY-MODE PULSATORS AMONG SUBDWARF B STARS: PG 1716+426, THE CLASS PROTOTYPE

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## ABSTRACT

A new class of pulsating subdwarf B stars has recently been announced by Green and coworkers. Here we present a follow-up paper describing our observations and the pulsation structure of the class prototype PG 1716+426. The oscillations are multiperiodic with periods between 0.8 and 1.4 hr (180–340  $\mu$ Hz) and semiamplitudes less than 0.2%. We also observe that the periods and amplitudes appear variable, making the pulsation structure of PG 1716 complicated. The periods are an order of magnitude longer than those seen in EC 14026 (sdBV) stars, implying that they are gravity modes rather than pressure modes. As such, they represent a new class of variable star.

Subject headings: stars: individual (PG 1716+426) — stars: oscillations — subdwarfs

### 1. INTRODUCTION

Subdwarf B (sdB) stars are the field counterparts to extended horizontal-branch stars observed in globular clusters (Heber 1986). They are thought to be stars with masses of about 0.5  $M_{\odot}$  with thin (<10<sup>-3</sup>  $M_{\odot}$ ) hydrogen shells and temperatures from 22,000 to 40,000 K (Saffer et al. 1994), making them exceedingly blue.

Pulsating sdB stars with periods of a few minutes (EC 14026 stars after that prototype, but commonly referred to as sdBV stars) were first observed by Kilkenny et al. (1997) nearly simultaneously to their predicted existence by Charpinet et al. (1996, 1997). The EC 14026 stars have pulsation periods ranging from 90 to 600 s with amplitudes typically near 1%. The pulsations are likely driven by the  $\kappa$  mechanism, because of a diffusive Fe opacity bump in the envelope (Charpinet et al. 1997). EC 14026 pulsators are typically found among the hotter sdB stars, with  $T_{\rm eff} \sim 34,000$  K and  $\log g \sim 5.8$ . Two

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nice reviews of this class of pulsators are Kilkenny (2001) for an observational summation of over 30 class members and Charpinet, Fontaine, & Brassard (2001) for a description of the pulsation mechanism.

Green et al. (2003) announced the discovery of a new class of variable sdB stars with periods significantly longer than the EC 14026 class. These variables have periods from 45 minutes to 2 hr with amplitudes typically less than 1 millimagnitude (0.1%). To date, 13 members have been discovered with effective temperatures of ~25,000-30,000 K.

PG 1716+426 (hereafter PG 1716) was identified as an sdB star in the Palomar-Green survey (Green, Schmidt, & Liebert 1986). Multicolor photometry (Wesemael et al. 1992) and low-resolution spectra (Saffer et al. 1994) confirm that PG 1716 is a moderately cool sdB star, although the exact temperature is somewhat model-dependent. Saffer et al. (1994) derived  $T_{\text{eff}} = 27,400$  K,  $\log g = 5.47$  by fitting the Balmer lines with zero-metal atmospheric models. Morales-Rueda et al. (2003) found  $T_{\text{eff}} = 26,100$  K and  $\log g = 5.33$ using 1.4 Å resolution spectra and metal line-blanketed models. U. Heber (2003, private communication) has fitted Saffer's spectrum with both a low-metallicity grid ([M/H] =-2.0) and the same solar-metallicity one used by Morales-Rueda et al. (2003) to fit PG 1716. He determined that  $T_{\rm eff} =$  $27,527 \pm 119$  K and  $\log g = 5.42 \pm 0.02$  (low-metallicity case) and  $T_{\rm eff} = 26,541 \pm 115$  K and  $\log g = 5.40 \pm 0.02$ (high-metallicity case), in fairly good agreement with Saffer et al. (1994) and Morales-Rueda et al. (2003), respectively. Figure 1 shows a very high signal-to-noise ratio, 1 Å resolution spectrum (E. Green et al. 2004, in preparation), but the small number of Balmer lines appears to introduce systematic offsets in temperature and gravity that are not completely understood.

#### 2. CCD PHOTOMETRY

The discovery light curve was obtained on the 2.3 m Steward telescope at Kitt Peak while searching for eclipses, reflection, and/or ellipsoidal effects in sdB stars. Additional CCD photometry was obtained at Mount Bigelow, the Nordic Optical Telescope (NOT), Calar Alto, and Fick Observatories. The specific runs are listed in Table 1.

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Fig. 1.—High-resolution MMT spectrum of PG 1716 (E. Green et al. 2004, in preparation).

The majority of the data were obtained on the Mount Bigelow 1.55 m using a conventional 2K CCD binned  $3 \times 3$ and an *R* filter. Integration times were 60 s with an overhead time of 24 s. Magnitudes for the target star and comparable reference stars in each image were derived using aperture photometry. Extensive tests show that our photometric errors are minimized when the radius of the star aperture is about 2.5 times the stellar FWHM, the inner radius of the sky annulus is 4.0 times FWHM, and the outer-sky radius is chosen so that the sky area is 4 times the area of the star aperture.

The Nordic data were obtained using the real time photometry (RTP; Østensen & Solheim 2000) program, which frame-transfers  $64 \times 64$  pixel images. 12 s integrations were combined with ~5 s read times to give a complete cycle time

TABLE 1Observations of PG 1716

Run	Length (hr)	HJD (2,452,000+)	Observatory
mtb01	4.9	27.7726143	Mt. Bigelow
mtb02	4.9	28.7670177	Mt. Bigelow
mtb03	5.0	30.7691217	Mt. Bigelow
not01	6.1	34.4915752	Nordic
mtb04	5.7	44.7404914	Mt. Bigelow
ca01	2.7	45.5778145	Calar Alto
mtb05	7.4	45.6676902	Mt. Bigelow
ca02	1.1	46.6394816	Calar Alto
mtb06	7.1	46.6798948	Mt. Bigelow
mtb07	4.7	47.7835350	Mt. Bigelow
mtb08	4.6	50.7828577	Mt. Bigelow
mtb09	7.2	58.6762118	Mt. Bigelow
mtb10	7.0	73.6814992	Mt. Bigelow
mdr184	3.3	84.6369626	Fick 24 inch
mtb11	3.0	84.8543145	Mt. Bigelow
mdr185	3.4	85.6329429	Fick 24 inch
mdr188	1.9	87.6781068	Fick 24 inch
mtb12	6.9	89.6847894	Mt. Bigelow
mdr192	0.7	90.6237724	Fick 24 inch

Note.—HJD indicates the run start times and the Length is the entire run length, which can differ slightly from the light curves shown in Fig. 2 from which noisy data were removed.



Fig. 2.—Light curves for the useful data obtained for PG 1716. Each panel is 0.55 day with HJD given in the margin.

of  $\sim 20$  s per integration. However, the integrations were cycled through 3 filters (*V*, *R*, and *I*) in such a way as to provide an *R* frame every 40 s and one *V* and *I* frame every 80 s (i.e., *RVRIRV*...).

The Fick Observatory 24 inch (0.6 m) data were obtained using an Apogee Ap7 512  $\times$  512 CCD with no filter. Integration times of 15 s were used with the CCD binned 2  $\times$  2 and a read time of 4 s. We also used the Fick and NOT data to search for short-period variations. The last two Fick runs were not used in our analysis, as they were too short to be useful. Calar Alto data were obtained on the 2.2 m telescope using the BUSCA CCD photometer (Reif et al. 1999; Falter et al. 2003) with a 60 s read time and 20 s integrations. Light curves (*R* band for NOT and Calar Alto data) are shown in Figure 2.

In total, 87 hr of data were obtained over 64 days. Differential photometry was obtained from stars in the same field that were used to correct for extinction and sky variations. Additional color differences between stars were corrected using low-order polynomials. With pulsations on the same order as typical transparency changes, reductions can be quite complicated. Our philosophy here has been to remove trends that are obvious over several pulsation cycles (>3 hr) and leave those that are not, as only correlated signal will contribute to peaks in the temporal spectrum (Fourier transform; FT), while uncorrelated signal increases the "noise."



Fig. 3.—Temporal spectra of varying amounts of data. Inset in each panel is the data window to scale, and the inclusive Julian dates (from 2,452,000.0) are given at the right. The data for dates 27-58 were used in the accompanying analysis.

#### 3. ANALYSIS

We follow the standard procedure for determining pulsation frequencies from time-series photometry (i.e., Winget et al. 1991). From individual runs, we determine in what range the pulsations are. Of course, with periods near an hour, they are too long for a single site to observe more than a handful of cycles per night. As such, it is necessary to combine temporally adjacent runs to look for pulsations. One way to help discern pulsation from aliasing is to examine several combinations of data and presume that peaks common to the data sets that are not located on obvious aliases are real. Figure 3 shows the combinations we used in our analysis, as well as the data windows. The data windows are a noise-free sine wave sampled at the same times as the data. Thus, the main window peak is the input frequency, and every other peak is an alias that should be included in the real data as well. The FT in the second panel of Figure 3 was used for our reductions, as it represents the best time resolution without adding too much noise from aliasing. The other two individual data sets were used as a check.

Another complication is amplitude variation, which can be caused by beating with the periods of other pulsations, unresolved pulsations, or variations intrinsic to that particular mode.<sup>12</sup> The result is that amplitude variations can add to an already complex alias pattern. To evaluate any amplitude variations, we examined the temporal spectra of individual runs. Fourier transforms are provided in Figure 4 and show that the peaks shift on timescales of a day or shorter. This means that alias peaks will appear in our temporal spectrum on the timescale of amplitude variation. Unfortunately, our coverage is not sufficient to effectively determine this timescale.

Examining Figures 2, 3, and 4, there are a few obvious remarks that need to be made as a forewarning on the remaining analysis. PG 1716 is clearly a variable star and multiperiodic. The pulsation amplitudes are extremely low (<2 mmi; millimodulation intensities) and likely variable, and the periods are on the same scale as normal-sky variations, which makes reducing the noise in the FT difficult. As such, presumably real pulsation peaks in the temporal spectrum are



Fig. 4.—Temporal spectra of data runs from Fig. 2 and Table 1. Dashed vertical lines indicate the highest amplitude, longest, and shortest frequencies (283, 349, and 183  $\mu$ Hz, respectively) and are merely for clarity. It is obvious that the pulsation amplitudes, and probably the frequencies, are changing with time.

dangerously close to the noise, and the alias pattern is very single-site–like in appearance. The only concrete statement we can make is that PG 1716 *is* a variable star with multiple periods between 180 and 350  $\mu$ Hz (0.8–1.5 hr). Although we will proceed with our analysis, it is with the note that specific periods may not be well determined because of the complexity of the data. It is our hope that this analysis will motivate further observations to obtain better data (i.e., more complete coverage) to confirm or modify our periods.

To best determine the periods in the data, we used the standard techniques of least-squares fitting and prewhitening. We began by nonlinear least-squares fitting of the frequencies, amplitudes, and phases of the peaks that appear above the noise and are common to all panels of Figure 3. This first step is shown as the difference between the top two panels of Figure 5, from which we detected three periods. The temporal spectrum has four more substantial peaks, and although they are not obvious in the original FT, they do appear in regions of power not included in the first three periods. After prewhitening of all six periods, the temporal spectrum does not show any significant peaks and is below 0.5 mmi (shown in Fig. 5 with arrows for all six prewhitened frequencies; the bottom panel is the spectral window for comparison). The six periods, with their amplitudes and formal errors, are given in Table 2. As a check, we again fitted the frequencies found from our working data set to various data sets with inclusive Julian dates of 2,452,027-2,452,089 (all the useful data),

<sup>&</sup>lt;sup>12</sup> Many EC 14026 class members show intrinsic amplitude variations.



Fig. 5.—Temporal spectra showing the prewhitening of identified pulsations. The top panel shows the original data, while the next two panels are prewhitened by three and six periods, respectively (*arrows*). The bottom panel shows the data window.

2,452,044–2,452,050, 2,452,027–2,452,034, and 2,452,073– 2,452,089. We were able to recover all six periods in all data sets. Our best-fit periods contain three groups of frequencies with peaks separated by one (11.5  $\mu$ Hz) or three (34.5  $\mu$ Hz) daily aliases. The best conclusion would thus be that the data really contain three periods, and that amplitude variation or temporal aliasing is responsible for the other three. We also examined our data for short-period (EC 14026–type) pulsations. Both the Fick and NOT data would be sensitive to short-period (a few minutes) pulsations. The Fick data were extremely noisy and not of use, but the NOT data were excellent for such a test. Figure 6 clearly shows that no shortperiod pulsations are present in the data to a limit of better than 0.04%.

## 4. RESULTS

As periods detected in this pulsation class are substantially longer than the sound-crossing time for sdB stars, they should all be *g*-mode pulsations (Fontaine et al. 2003). In the limit  $n \gg l$ , *g*-modes obey the asymptotic relation

$$\Pi_{n,l} \approx \frac{\Pi_0}{\sqrt{l(l+1)}} \left(n+\epsilon\right)$$

(Smeyers & Tassoul 1987), where *n* and *l* describe the spherical harmonics,  $\Pi_{n,l}$  is the observed period, and  $\Pi_0$  is a constant that depends on an integral of thermodynamic

TABLE 2 Pulsation Periods Detected

ID	Period (s)	Frequency (µHz)	Amplitude (mmi)
f1	2939.39(13)	340.207(15)	1.24(10)
f 2	3034.21(16)	329.575(17)	1.10(10)
f 3	3522.09(15)	283.922(12)	1.51(9)
f4	3665.85(22)	272.788(16)	1.10(9)
f 5	4573.25(31)	218.663(15)	1.11(9)
f6	5459.78(46)	183.158(15)	1.06(9)

Note.—Formal least-squares errors given in parentheses. Amplitudes are measured in millimodulation intensities (mmi).



FIG. 6.—Temporal spectra of the Nordic data. Left and bottom right panels are plotted on the same vertical scale, while the data window (*top right*) is plotted at an arbitrary scale. The left panel shows the frequency range containing all the pulsations, while the bottom right panel shows the frequency range of the EC 14026 class pulsators. The data window is plotted on the same horizontal scale as the left panel.

quantities over the structure of the star and is expected to be around 200 s for sdB stars. This is well short of the periods we are considering. As such, the asymptotic approximation should be realized in PG 1716–class stars, and pulsations of successive *n* but same *l* should be evenly spaced in *period*. We note that for *p*-modes, the asymptotic limit becomes

$$\nu_{n,l} \approx \nu_0 \left( n + \frac{l}{2} + \epsilon \right),$$

which results in successive *n*-modes of the same *l* that are equally spaced in frequency. Figure 7 compares pulsation periods in a representative evolutionary model (log g = 5.56,  $T_{\rm eff} = 26,000$  K) to those observed. No in-depth model search was performed, yet the results are instructive. As expected, the periods are nearly equally spaced for all *l* (we calculated through l = 4) values. The most important feature is the



FIG. 7.—Comparison between observed periods and a model of appropriate  $T_{\rm eff}$  and log g.



Fig. 8.—Temporal spectra in frequency and time (s). Frequency and period spacings between pulsations are indicated and are connected by a dashed line.

*number* of model periods to those observed. Within the observed period range (2800–5500  $\mu$ Hz), the model produces 112 separate, nonrotationally split m = 0 periods. With so many model periods in the observed range, additional clues (such as rotationally split multiplets or time-series spectroscopy) will likely be necessary to match observations with models.

Under further scrutiny, a couple of interesting patterns are found in our determined frequencies. Figure 8 shows the temporal spectrum, with arrows indicating our best-fit frequencies and periods. Under the asymptotic limit previously described, we would expect to see nearly equally spaced periods (*bottom*). Three of our periods are split nearly equally in period (by 908 and 886 s), which may indicate that they have the same *l*-value. However, they are too far apart to have consecutive *n*. We also detect three modes (two in groups) equally spaced in *frequency* (*top*). The smaller splittings are almost certainly complications caused by daily aliasing (11 and 10  $\mu$ Hz, respectively), but the spacings between these groups is nearly the same (54 and 57  $\mu$ Hz, respectively). Stellar rotation can create equally spaced modes in frequency separated by

$$\delta\nu_{n,l} = \frac{\left(1 - C_{n,l}\right)}{\Pi_{\rm rot}}$$

where  $\Pi_{rot}$  is the rotation period of the star and  $C_{n,l}$  is a small correction (typically a few percent in sdB stars). If indeed these were consecutive *m*-modes, it would imply a rotation period of 4.2 hr. However, PG 1716 is a member of a known binary about an unseen, probable white dwarf companion with a relatively short orbital period (P = 1.777373 days or  $\sim$ 42.7 hr; E. Green et al. 2004, in preparation; P = 1.77732days, Morales-Rueda et al. 2003). Interestingly, the orbital period is about 10 times the period spacing. However, such a close binary would have evolved through a common-envelope phase, likely tidally locking the sdB to the companion. We raise the possibility that either the sdB star is not tidally locked or these could be p-modes of consecutive n or l (although we have no idea how this could occur for such long periods). Clearly there is much more to learn from PG 1716, but it will require substantially better temporal coverage to do it.

#### 5. CONCLUSIONS

Serendipity has again played its role in sdB star research. Much like the accidental discovery of EC 14026 (short-period pulsating sdB) stars (Kilkenny et al. 1997), the search for close companions has revealed a new class of pulsating sdB stars. Subsequent follow-up observations at several observatories using different detectors confirmed the variable nature. Green et al. (2003) report the detection of 20 class members already, with PG 1716 having been observed most frequently. With multiple class members, measurements with different instruments at different locations, and periods of variations not commensurate with the binary period, it seems safe to conclude that PG 1716 is the prototype for a new class of variable stars.

We detect up to six separate pulsation periods, with the most likely interpretation being three real modes split by daily aliases. The periods are between 0.8 and 1.4 hr with amplitudes between 0.7 and 1.6 mmi (0.07%–0.16%). However, our 70 hr of useful observations are stretched out over 58 days and are nearly all from the same longitude. From Figure 4, it is also likely that pulsation amplitudes and frequencies are changing. As such, the temporal spectrum is extremely complex, making unique period determinations impossible. Stars in the PG 1716 class would benefit greatly from multisite observations.

For periods longer than 45 minutes and substantially longer than the radial fundamental mode, high-order g modes are the best interpretation for the variations. As such, EC 14026 and PG 1716 sdB pulsators are analogous to  $\delta$  Scuti and  $\gamma$  Doradus main-sequence variables or the  $\beta$  Cepheid and slowly pulsating B stars. In all three cases, the hotter stars are p-mode pulsators, while the cooler ones are g-mode pulsators (Dziembowski, Moskalik, & Pamyatnykh 1993; Green et al. 2003).

Comparing a representative evolutionary model to the observations, we find that models have many more modes than those observed (112 compared to 6). This will make mode identification difficult at best. However, we also (marginally) detect equally spaced periods and frequencies. These spacings are most easily interpreted as modes of the same l values (for even period spacing) or as *m*-modes separated by the rotation frequency (for even frequency spacing). The frequency spacing we measure would imply a rotation period of 4.2 hr, which does not agree with the orbital period of nearly 1.8 days. For such a post–common-envelope close binary not to be tidally locked seems unlikely. These factors combine to make PG 1716 a very interesting star, and we look forward to further observations clarifying some the issues we have raised. There is yet much to learn from PG 1716.

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